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LETTER TO THE EDITOR

The self-avoiding walk on the honeycomb lattice

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Abstract. We analyse the chain generating function for the self-avoiding walk problem on the honeycomb lattice. We use the exact critical point value of Nienhuis and find exponent values of $\gamma \sim 1.344$ and $\Delta_1 \sim 0.93$, for the dominant and first confluent exponents. The former value is in agreement with Nienhuis' exact result. Second and third confluent exponents, $\Delta_2 \sim 1.2$ and $\Delta_3 \sim 1.5$, are also identified, these exponents being consistent with Monte Carlo results and Nienhuis' exact irrelevant eigenvalue, respectively.

There is at present much interest in the literature concerning the nature of correction-to-scaling terms. We consider below corrections to the dominant critical behaviour in the self-avoiding walk problem; this work has been stimulated by Nienhuis' (1982) recent exact results for the O(n) model in two dimensions.

In this letter we investigate the behaviour of the chain generating function $C(t)$ for the self-avoiding walk on the honeycomb lattice and hypothesise

$$C(t) \sim (t_c - t)^{-\gamma}(1 + a_1(t_c - t)^{\Delta_1} + a_2(t_c - t)^{\Delta_2} + a_3(t_c - t)^{\Delta_3} + \ldots)$$

to obtain some interesting conclusions.

We study the 34-term series of Sykes *et al* (1972) for this quantity using the method of Adler *et al* (1982, 1983). This method involves transforming a series in t to one in y where

$$y = 1 - (1 - t/t_c)^{\Delta}$$

and then studying the function

$$F_\Delta(y) = C(t(y)) \sim \text{constant}\ t_c^{-\gamma}(1-y)^{-\gamma/\Delta}(1 + a_1 t_c^{\Delta_1}(1-y)^{\Delta_1/\Delta} + \ldots)$$

by taking Padé approximants to the function

$$\gamma_{\text{out}}(\Delta) = \{\Delta(1-y)[\mathrm{d}(\ln(F_\Delta(y))/\mathrm{d}y)]\}_{y=1}.$$

Different approximants define a family of $\gamma = \gamma_{\text{out}}(\Delta)$ curves in the (Δ, γ) plane. We believe that at the correct values of γ and t_c and at least one of the Δ_i, these curves will intersect (Adler *et al* 1982, 1983). There may be intersection regions for more than one Δ_i value; however, these will not necessarily occur at the correct γ value. In fact we identify the *first* non-analytic confluent correction as the *first* Δ_i value for which such an intersection occurs, unless a certain ratio between Δ_i values is observed and all the Δ_i give the same γ (see below). Higher Δ_i values should, in theory, also give the correct γ value; however, the lower Δ_j (where $j < i$) term may lead to a systematic error in γ. Similarly, the higher Δ_i terms may lead to systematic errors in the evaluation of Δ_1 and it is by now well established that the Δ_1 term can lead to systematic errors in evaluating γ (see the review in Adler *et al* (1983)). The situation noted above, where there are several intersections at the same γ_i value and where the Δ values can be identified as Δ_1 and Δ_1/k where $k = 1, 2 \ldots$, has occurred for test series (Privman 1983a) and certain models whose exponents are known exactly (Adler and Enting 1983, Adler 1983).

In figure 1 we show a family of curves for the series of Sykes *et al* (1972). We observe intersection regions near $\Delta \sim 0.93$, (region A), 1.25, (region B), and 1.55, (region C), and indicate the exact γ value with an asterisk.

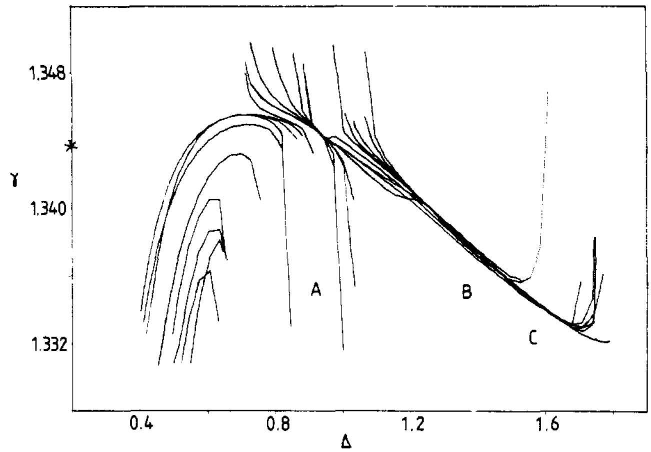

Figure 1. Curves of γ as a function of Δ for different Padé approximants. Three different regions of convergence are identified and these are denoted by the letters A, B and C. The A region may be an analytic term, and the other two regions are consistent with previously obtained correction terms. For this figure we used the exact $t_c = 1/(2+\sqrt{2})^{1/2}$ and indicate the exact γ value with an asterisk.

We may claim a γ value in agreement with Nienhuis' exact result that $\gamma = 1.343\,75$ only near the A region. In the B region, where $\Delta \sim 1.25$, in close agreement with the Monte Carlo value of 1.2 ± 0.1 and the ε-expansion result of 1.15 (Le Guillou and Zinn–Justin 1980), γ is not in good agreement with Nienhuis' conjecture. In the C region, where Δ is consistent with Nienhuis' exact result for an irrelevant eigenvalue, namely 1.5, the value is even further from the conjectured exact value. It thus appears that neither the B or C region is the correct first non-analytic correction Δ_1.

This leaves us with the A region. Here Δ is very close indeed to 1.0 and thus this region could be an analytic correction. If this is the case then the first non-analytic correction must have a very small amplitude (a_1) on the honeycomb lattice, and presumably lies somewhere between 0.5 and 1.1, since the conjectured γ value is only obtained within this range. This latter possibility is supported by the evidence of Privman (1983b), Guttmann (1983) and Djordjevic *et al* (1983).

In this work three different regions, which correspond to three different confluent exponents, have been identified. All these exponents have been previously observed, but to the best of the author's knowledge no other single calculation has indicated the presence of all three of them contemporaneously.

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